30 Polynomials with Real Zeros

Due:
12/14/2015 at 06:00am EST.

Students will be able to:

- Use Rational Zero Theorem to find zeros of polynomials
- Determine the maximum possible number of positive real zeros and negative real zeros of a polynomial
- Factor a polynomial using Factor Theorem
- Use Synthetic Division to divide polynomials

Functions and symbols that WeBWorK understands.

Links to some useful WeBWorK pages for students

1. (1 pt) Find all rational zeros of the polynomial

$$P(x) = 4x^4 - 12x^3 - 12x^2 - 12x - 16$$

Its rational zeros are $x_1 =$____, $x_2 =$____, $x_3 =$____ and $x_4 =$____ with $x_1 \le x_2 \le x_3 \le x_4$
If the polynomial has only three rational zeros, input them at x_1, x_2 and x_3; If the polynomial has only two rational zeros, input them at x_1 and x_2; if the polynomial has only one rational zero, input it at x_1.

2. (1 pt) Find all the real zeros of the polynomial

$$P(x) = x^3 - 4x^2 - 13x + 6$$

Its real zeros are $x_1 =$____, $x_2 =$____ and $x_3 =$____ with $x_1 \le x_2 \le x_3$
If the polynomial has only two real zeros, input them at x_1 and x_2; if the polynomial has only one real zero, input it at x_1.

3. (1 pt) For the function $y = x^5 + 5x^3 - 24x$, find all real zeros.
Note: If there is more than one real zero, separate the answers by commas. Also, if you want to enter the square root of a number, like two, enter sqrt(2).
The real zeros are $x =$_____.

4. (1 pt) List all possible rational roots for the function

$$f(x) = 5x^4 - 4x^3 - 5x^2 + 9x + 55.$$

Give your list in increasing order. Beside each possible rational root, type "yes" if it is a root and "no" if it is not a root. Leave any unnecessary answer blanks empty.

Possible rational root: __ Is it a root? __.
Possible rational root: __ Is it a root? __.
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5. (1 pt) Give all of the zeros of the polynomial

$$P(x) = x^3 - 3x^2 - 7x - 15.$$

as a comma separated list.

6. (1 pt) $f(x) = x^8 + 11x^7 + 20x^6 - 566x^5 - 5104x^4 - 13200x^3 + 9216x^2 + 61152x^1 + 43264$

What is the maximum number of positive real roots for $f(x)$?
____
What is the maximum number of negative real roots for $f(x)$?
____

7. (1 pt) Find all rational zeros of the polynomial

$$P(x) = 4x^4 + 9x^3 - 5x^2 + 9x - 9.$$

Give a comma separated list of the rational zeros. If there are no rational zeros, enter the word *none* .

8. (1 pt) Factor $P(x) = x^3 + 5x^2 + 8x + 16$ into linear and irreducible quadratic factors with real coefficients.
Let $P(x) = (x+a)(x^2+bx+c)$. Then
$a =$____
$b =$____
$c =$____

9. (1 pt) Use the Factor Theorem to show that $x - 1/2$ is a factor of

$$P(x) = 2x^3 - 9x^2 + 8x - 2.$$

The function value $P(1/2) =$ ____.
Thus, $x - 1/2$ is a _____ of $P(x)$.

10. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^4 + 7x^3 + 3x^2 + 25x + 35, \quad c = -7.$$

The quotient is _____
The remainder is _____
$P(c)$ = _____

11. (1 pt) Use the Factor Theorem to show that $x - 1$ is a factor of

$$P(x) = x^3 - 7x^2 + 14x - 8.$$

The function value $P(1) =$ _____.
Thus, $x - 1$ is a _____ of $P(x)$.

12. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^2 + 2x + 2, \quad c = -1.$$

The quotient is _____
The remainder is _____
$P(c)$ = _____

13. (1 pt) Use synthetic division and the Remainder Theorem to evaluate $P(c)$, where

$$P(x) = x^3 - 6x^2 + 9x - 5, \quad c = 2.$$

The quotient is _____
The remainder is _____
$P(c)$ = _____